

Common Core State Standards Grades K-2 Mathematics

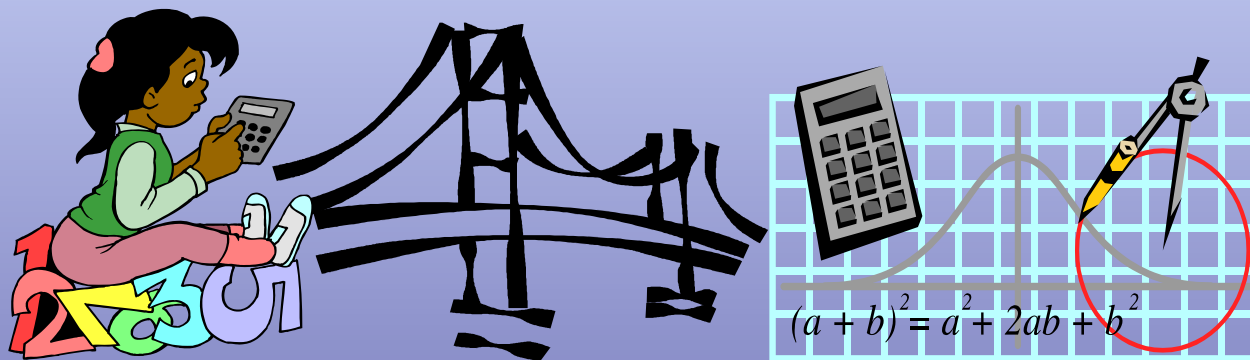
Webinar #1: Addition & Subtraction (Operations and Algebraic Thinking Domain)

Presented by the Office of Curriculum, Instruction and Student Support

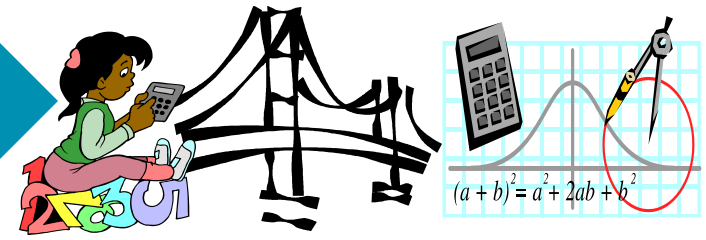
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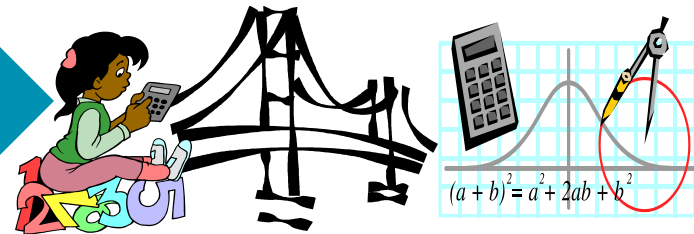
What we hope you'll walk away with



Our Intentions:

- *Increased understanding of the progression of addition and subtraction expectations in grades K-2 (and a little beyond).*
- *Awareness of available resources for teachers to access on the Standards Toolkit website.*

Important “Shifts” Compelled by the CCSS for Mathematics



1. *Focus and Coherence*

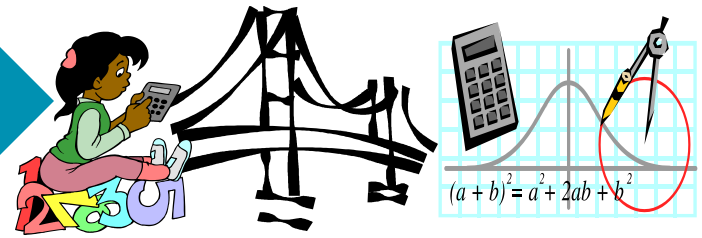
2. *Progression towards Fluency*

3. The “Understand” Standards

4. The Standards for Mathematical Practice

- a. Problem-solving and modeling
- b. Habits-of-mind; productive disposition
- c. Communication
- d. Reasoning and Sense-making

Teaching to the Big Ideas in the CCSS

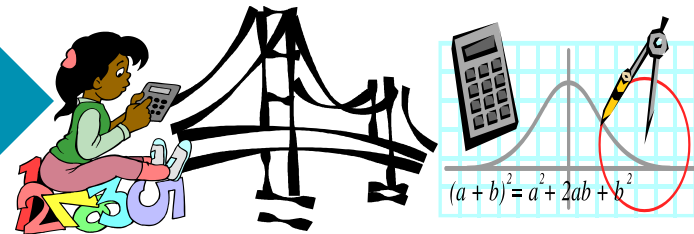


Domain: Operations and Algebraic Thinking

Clusters:

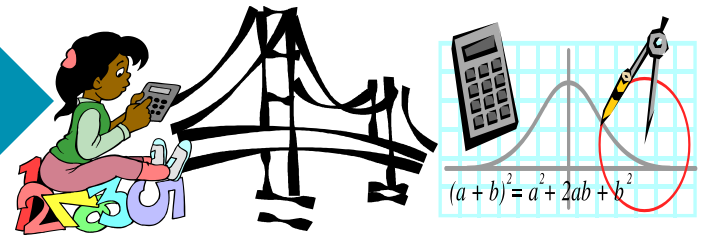
- *Understand addition as putting together & adding to, and understand subtraction as taking apart & taking from. (K)*
- *Represent and solve problems involving addition and subtraction. (1 & 2)*
- *Add and subtract within 20. (1 & 2)*
- *Work with addition and subtraction equations. (1)*
- *Work with equal groups of objects to gain foundations for multiplication. (2)*

Important “Shifts” Compelled by the CCSS for Mathematics



1. Focus and Coherence
2. Progression towards Fluency
3. ***The “Understand” Standards***
4. The Standards for Mathematical Practice
 - a. Problem-solving and modeling
 - b. Habits-of-mind; productive disposition
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“Understand” standards



1.OA.7: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

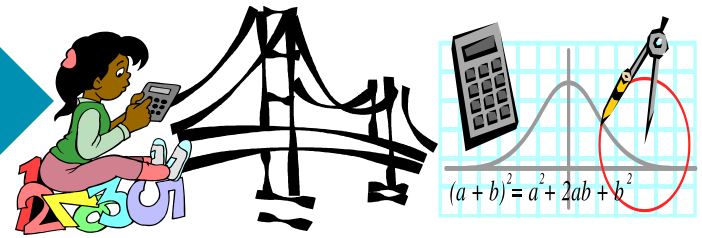
What does “=”
mean?



The equal sign?
Equals means
equals.



What do we expect
students to understand?



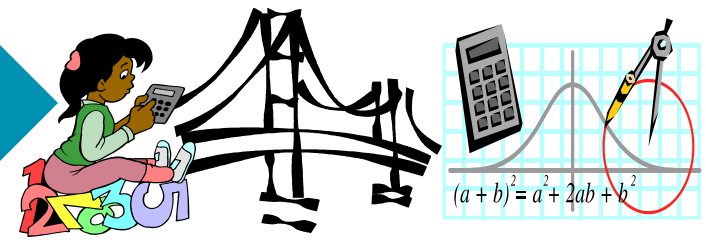
What does the equal sign really mean?



• It states a relationship between two quantities

- If two quantities are equal, they represent the same amount
- **EQUALS** means ***“THE SAME AS”***

Putting their understanding to good use

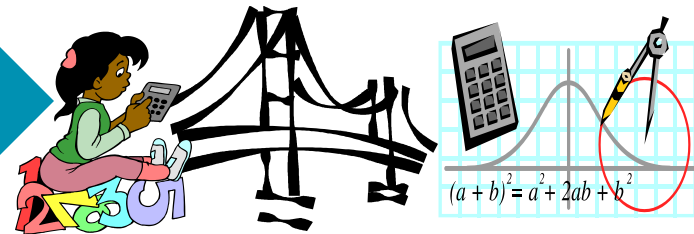


The **actions** we perform on the concrete and pictorial representations should have a direct connection to the abstract representation that we want students to understand and develop expertise with.

A diagram illustrating the equation $4 = 1 + 3$ using concrete representations. On the left, a large light blue oval contains four blue circles. In the middle is an equals sign. On the right, a large light blue oval contains one blue circle followed by three red circles. Below the ovals, the equation $4 = 1 + 3$ is written in large black numbers.

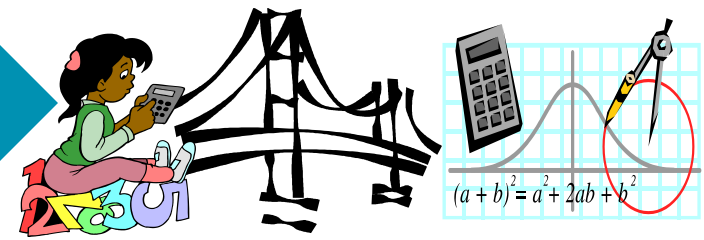
$$4 = 1 + 3$$

Important “Shifts” Compelled by the CCSS for Mathematics



1. Focus and Coherence
2. Progression towards Fluency
3. The “Understand” Standards
- 4. *The Standards for Mathematical Practice***
 - a. Problem-solving and modeling*
 - b. Habits-of-mind; productive disposition*
 - c. Communication*
 - d. Reasoning and Sense-making*

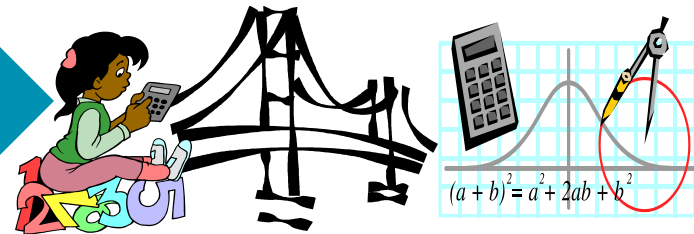
What we hope you'll walk away with



Our Intentions:

- *Increased understanding of the progression of addition and subtraction expectations in grades K-2 (and a little beyond).*
 - ◆ *Strategies for addition and subtraction*
 - ◆ *Fluency expectations for addition and subtraction*
 - ◆ *Word problems*
 - ◆ *Equality*
- *Awareness of available resources for teachers to access on the Standards Toolkit website.*

Focus and Coherence in the CCSS for Mathematics



Resource available at the Standards Toolkit Website:

→ Addition & Subtraction in the CCSS-M for Grades K-4

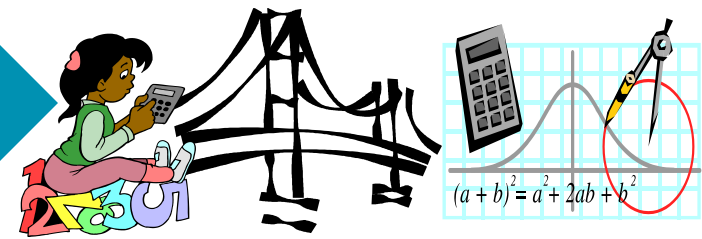
Addition and Subtraction in the CCSS-M for Grades K-4

A quick glance at the progression of expectations for complexity, fluency and representations



	<u>K</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
WORD PROBLEMS	K.OA.2: Solve addition & subtraction word problems to 10.	1.OA.1: Add & subtract word problems to 20 with unknowns in all positions. 1.OA.2: Word problems with 3 #s up to 20.	2.OA.1: Add & subtract within 100 to solve 1-2 step word problems with unknown in all positions	3.OA.8: Solve 2 step word problems using 4 operations with reasonableness	4.OA.3: Solve multistep word problems
<i>Complexity Progression</i>	<i>Simple</i>	<i>Unknowns in all positions</i>	<i>1- and 2-step</i>	<i>2-step</i>	<i>Multi-step</i>
<i>Concrete-Pictorial-Abstract Progression</i>	<i>Concrete and pictorial representations</i>	<i>Concrete, pictorial and abstract representations</i>	<i>Pictorial and abstract representations</i>	<i>Abstract representations</i>	<i>Abstract representations</i>

Strategies for Addition and Subtraction

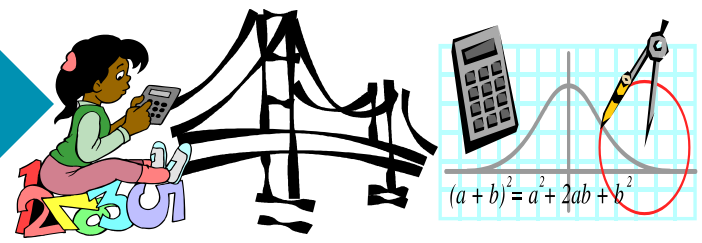


Addition and Subtraction in the CCSS-M for Grades K-4

A quick glance at the progression of expectations for complexity, fluency and representations

	K	1	2
STRATEGIES FOR ADDITION AND SUBTRACTION "C/P/A" refers to the types of REPRESENTATIONS students should work with "C" → Concrete Representations "P" → Pictorial Representations "A" → Abstract Representations (i.e., number sentences, expressions, equations)	<p>K.CC.4: Understand the relationship between numbers and quantities; connect counting to cardinality. (C/P)</p> <p>K.OA.1: Represent addition and subtraction with objects ... drawings ... expressions and equations (C/P/A)</p> <p>K.OA.3: Decompose numbers less than or equal to 10 into pairs in more than one way... (C/P/A)</p> <p>K.OA.4: For any number from 1 to 9, find the number that makes 10 when added to the given number. (C/P/A)</p> <p>K.NBT.1: Compose & decompose numbers from 11 to 19 into ten ones and some further ones ... (C/P/A)</p>	<p>1.OA.3: Apply properties of operations as strategies to add and subtract. (A)</p> <p>1.OA.4: Understand subtraction as unknown-addend problem. (C/P/A)</p> <p>1.OA.5: Relate counting to addition & subtraction. (C/P/A)</p> <p>1.OA.8: Determine the unknown number in an addition or subtraction equation relating three whole numbers. (C/P/A)</p> <p>1.NBT.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten (C/P/A)</p>	<p>2.NBT.6: Add up to four two-digit numbers based on place value and properties of operations.</p> <p>2.NBT.9: Explain why addition and subtraction work, using place value and the operations. (Explanations may be drawings or objects.) (C/P/A)</p> <p>2.NBT.3: Read & write #s to 1000 by using names and expanded form.</p> <p>2.NBT.7: Add and subtract within 1000, using models or drawings and strategies based on place value, properties of operations, relationship between addition and subtraction, and the relationship between a number and its place value. Understand that in adding or subtracting numbers, one adds or subtracts hundreds, tens and tens, ones and ones, and sometimes it is necessary to compose or decompose tens or hundreds. (C/P/A)</p> <p>2.OA.4: Use addition to find the total number of objects arranged in rectangular arrays up to 5 rows and up to 5 columns; write an equation to represent the total as a sum of equal addends.</p>
Recommended Strategies	<ul style="list-style-type: none"> Use 5-frames, 10-frames, dot cards and dice to help students quickly recognize pictorial representations of quantities (especially in relation to important benchmark numbers like 5 and 10) 	<ul style="list-style-type: none"> "Make Ten First" (for addition) or "Get to Ten First" (for subtraction). Break up a number (according to place value) and add up the parts → i.e., "partitioning" Add or subtract 10 (or a multiple of 10) → begin with "partitioning" then develop expertise with this to be able to do this fluently in one's mind. 	<ul style="list-style-type: none"> Break up a number (according to place value) "partitioning" Subtraction using "compensation" method Subtraction using regrouping (building to "regrouping"/"borrowing" algorithm by place value and subtracting the parts by place value) Associative Property (for sums of up to 100)

Strategies for Addition and Subtraction

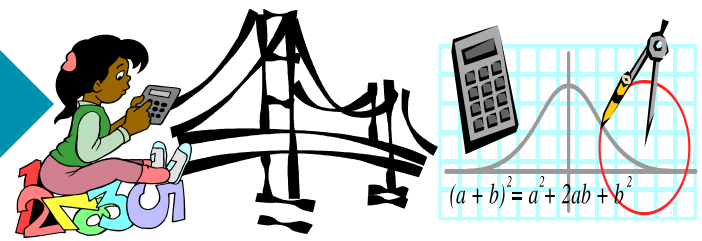


Grade K: up to 10

Grade 1: up to 100

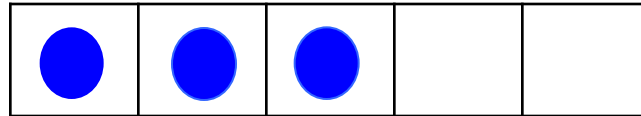
Grade 2: up to 1,000

Strategies for Addition and Subtraction

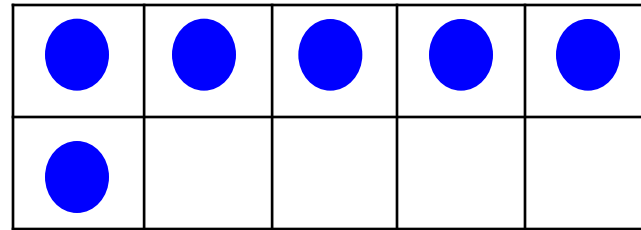


Grade K: Useful tools

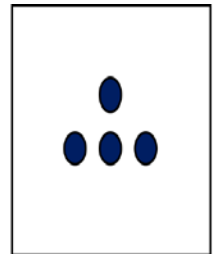
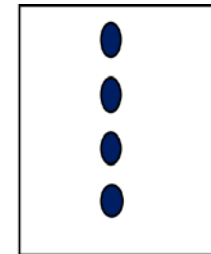
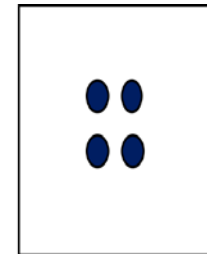
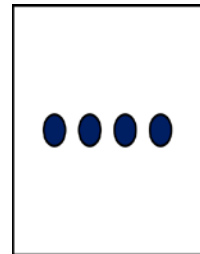
- 5 frames



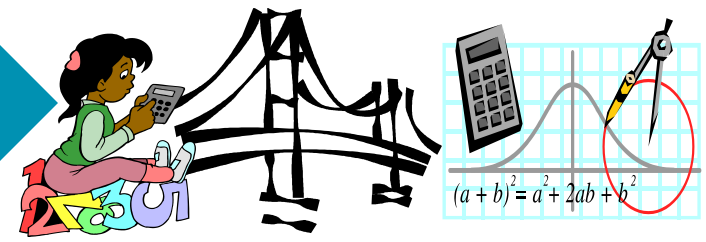
- 10 frames



- Dice and Dot Cards

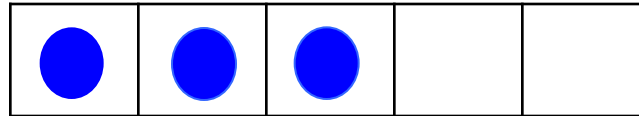


Strategies for Addition and Subtraction

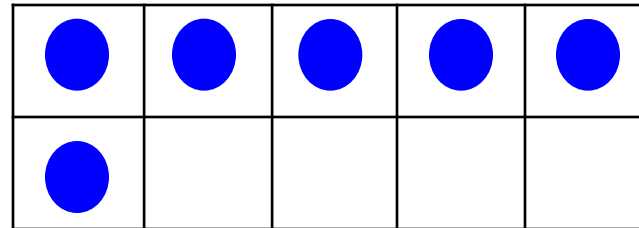


Grade K: Useful tools

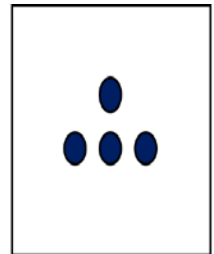
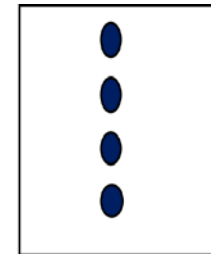
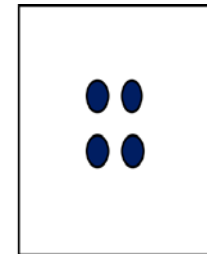
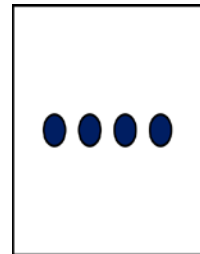
5 frames



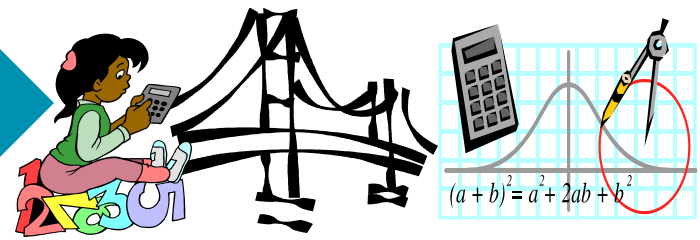
10 frames



Dice and Dot Cards



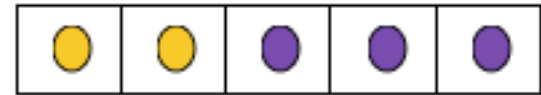
Strategies for Addition and Subtraction



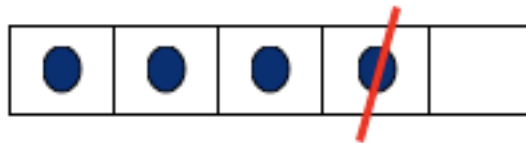
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



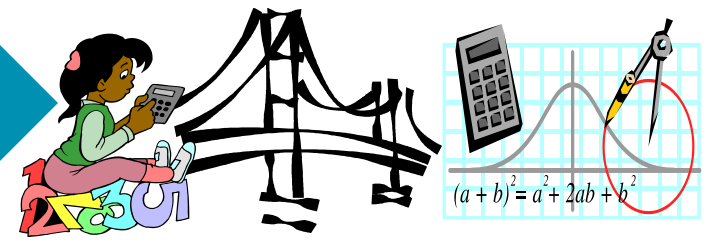
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

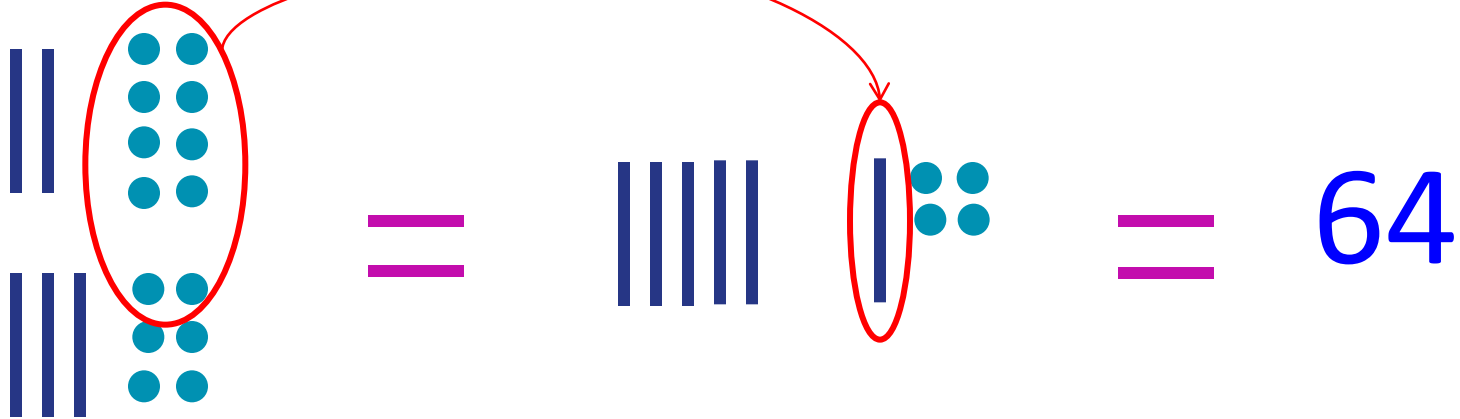


Strategies for Addition and Subtraction

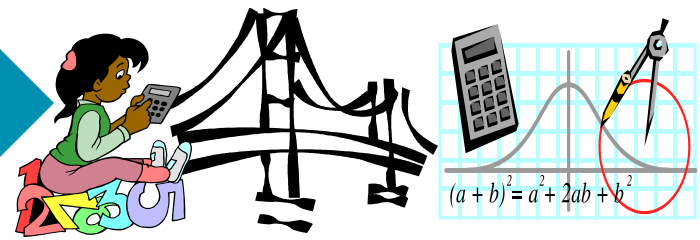


Strategic use of concrete and pictorial representations

$$28 + 36 =$$



Strategies for Addition and Subtraction



Find the sum of the numbers represented below.

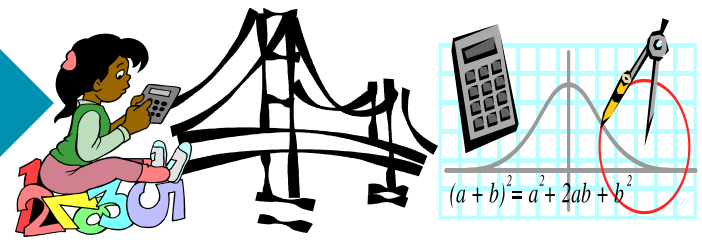
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Audience participation ... in the chat box please explain a strategy for determining the sum.

Strategies for Addition and Subtraction



Adding and Subtracting using Place Value (“partitioning”)

$$26 + 57 = 20 + 6 + 50 + 7$$

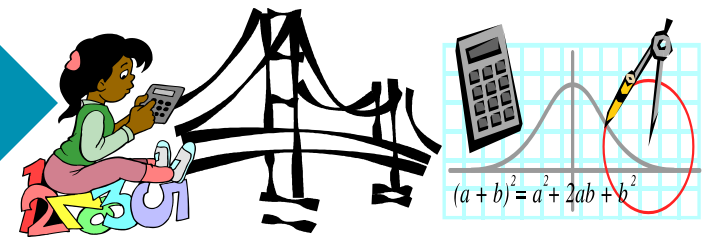
$$= 70 + 13$$

$$(= 70 + 10 + 3)$$

$$(= 80 + 3)$$

$$= 83$$

Strategies for Addition and Subtraction



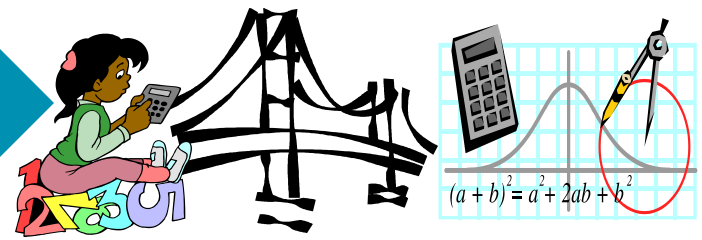
Show how to find each sum by “making ten” first.

a)	$7 + 5 = \square$
b)	$6 + 7 = \square$

Show how to find each sum by first getting to the nearest ten.

a)	$28 + 6 = \square$
b)	$34 + 59 = \square$

Strategies for Addition and Subtraction



Show how to find each sum by first getting to the nearest ten.

a) $28 + 6 = \boxed{}$

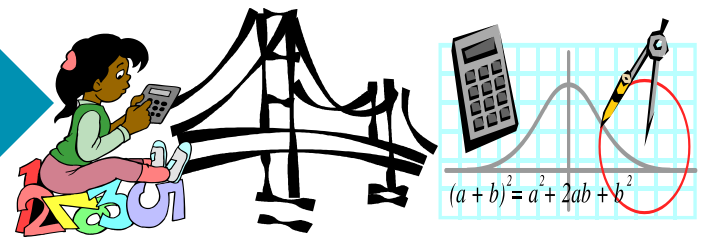
b) $34 + 59 = \boxed{}$

CAVEAT:

We must help students develop a cognitive map for deciding when to use a particular strategy. For example, consider

$$34 + 51 = \underline{\hspace{2cm}}$$

Strategies for Addition and Subtraction



Subtraction using the “COMPENSATION” method.

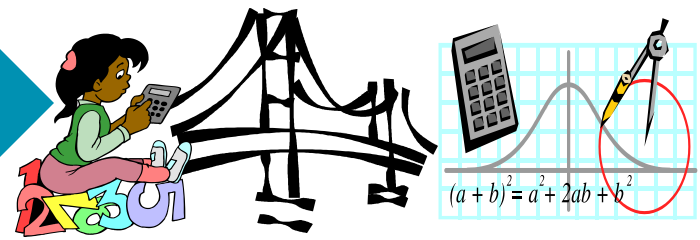
1. Make an equivalent expression where the smaller number has a zero in the ones place.

$$\begin{array}{r} 82 \\ - 34 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 88 \\ - 40 \\ \hline \end{array}$$

2. Make an equivalent expression where the larger number has a 9 in the ones place.

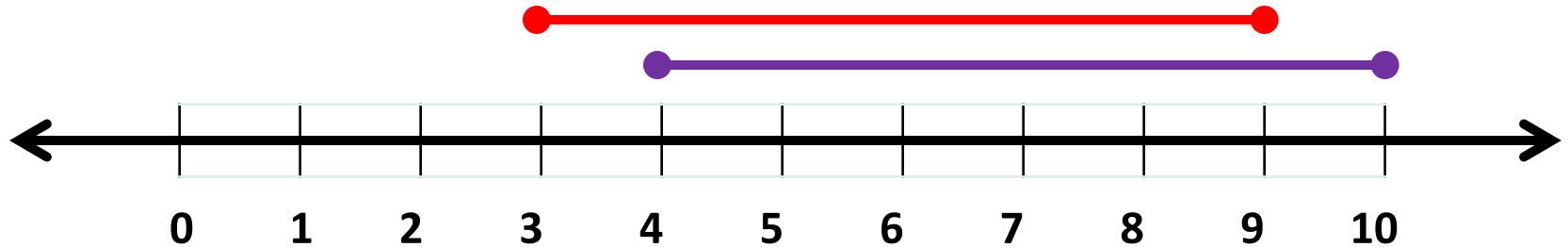
$$\begin{array}{r} 70 \\ - 46 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 69 \\ - 45 \\ \hline \end{array}$$

Strategies for Addition and Subtraction



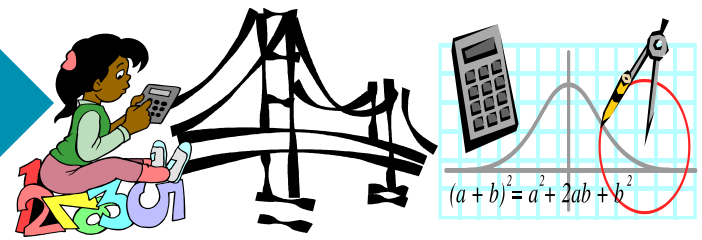
Subtraction using the “COMPENSATION” method.

One meaning of “subtraction” that we want students to understand is the idea of difference ... “10 – 4” is asking us, “What is the difference between 10 and 4?”

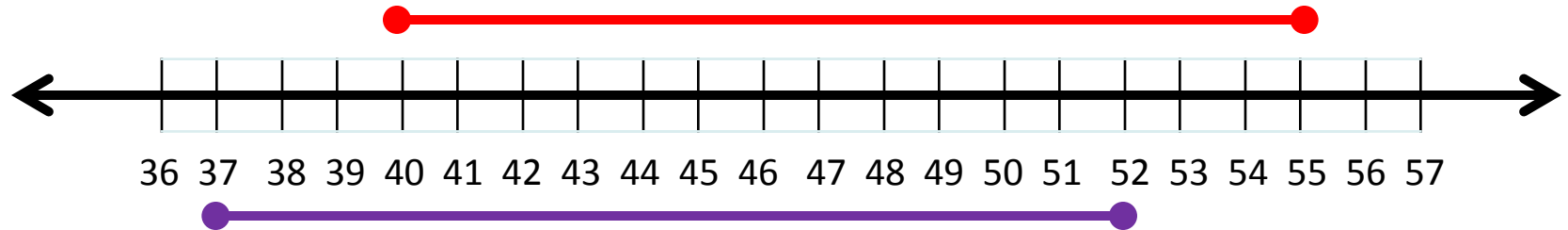


$$\begin{array}{r} 10 \\ - 4 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 9 \\ - 3 \\ \hline \end{array}$$

Strategies for Addition and Subtraction

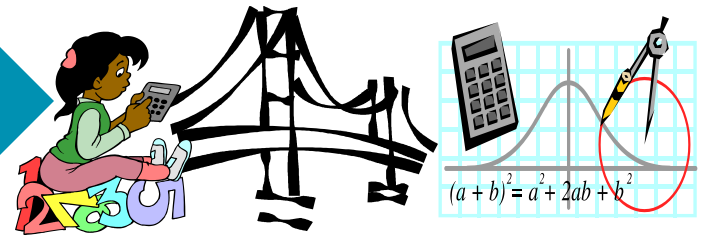


The “compensation method” for subtraction is not some kind of magic trick.



$$\begin{array}{r} 52 \\ - 37 \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 55 \\ - 40 \\ \hline \end{array}$$

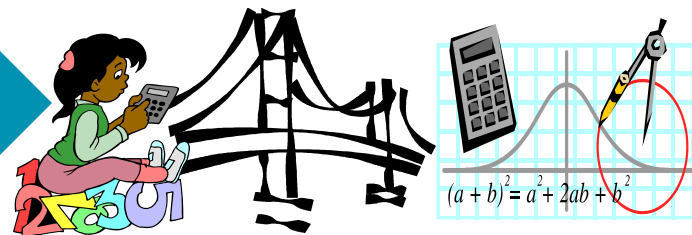
Clarification of Standard 1.NBT.4



1.NBT.4 – Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10...

- This standard is not limited to these instances.
- Make sure to include:
 - Double digit + double digit numbers

Fluency Expectations



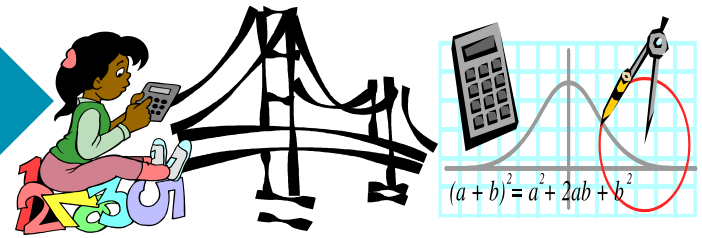
Addition and Subtraction in the CCSS-M for Grades K-4

A quick glance at the progression of expectations for complexity, fluency and representations

	<u>K</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
FLUENCY* WITH ADDITION AND SUBTRACTION	K.OA.5: Fluently add and subtract within 5.	1.OA.6: Add & subtract within 20, <i>demonstrating fluency for addition and subtraction within 10</i> . Use strategies ... 1.NBT.5: Use place value understanding and properties of operations to add and subtract. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. 1.NBT.6: Use place value understanding and properties of operations to add and subtract. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	2.OA.2: Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. 2.NBT.5: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 2.NBT.8: Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.	3.NBT.2: Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)	4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used.)
<i>Fluency Expectation</i>	<i>Up to 5</i>	<i>Up to 10</i>	<i>Up to 20 and then 100</i>	<i>Up to 1,000</i>	<i>Up to 1,000,000</i>

* Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

The Meaning of Fluency



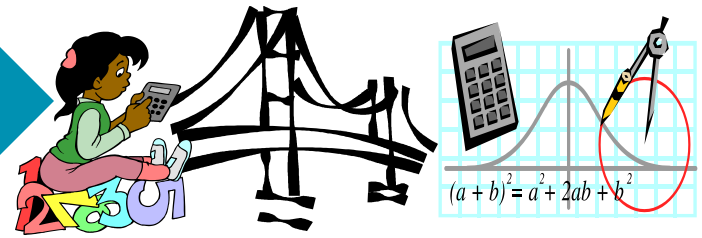
What does FLUENCY mean?

“Fluency is defined as ‘skill in carrying out procedures flexibly, accurately, efficiently and appropriately.’ – Jennifer Suh, Assistant Professor of Mathematics Education, George Mason University.

Fluency with basic addition facts can be defined as “the efficient, appropriate & flexible application of a single-digit calculation skills & is an essential aspect of mathematical proficiency.” - Arthur Baroody, Professor Emeritus of Curriculum & Instruction, University of Illinois

“A fluency approach to learning basic addition facts places a focus on developing and using mathematical strategies, with the goal of finding efficient, effective ways to apply known facts to derive unknown facts.” – Gina Kling, Professor Western Michigan University

Fluency Expectations



Fluency with Addition & Subtraction

Grade K: up to 5

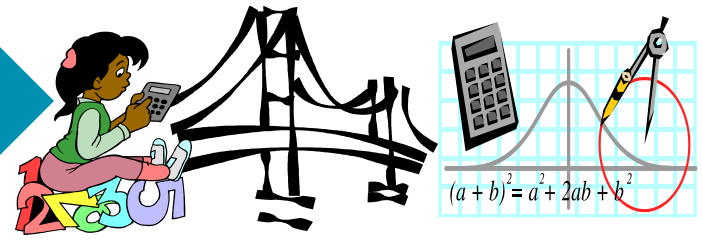
Grade 1: up to 10/20

Grade 2: up to 100

Grade 3: up to 1,000

Grade 4: up to 1,000,000

Difference between 2.OA.2 & 2.NBT.5



2.OA.2: Fluently add & subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.

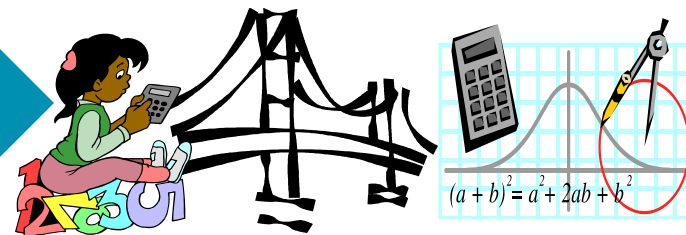
2.NBT.5: Fluently add & subtract within 100 using strategies...

Difference between the 2 standards:

2.OA.2 is about the mental math and moving toward the automaticity.

2.NBT.5 is about fluency and flexibility with strategies.

Word Problems

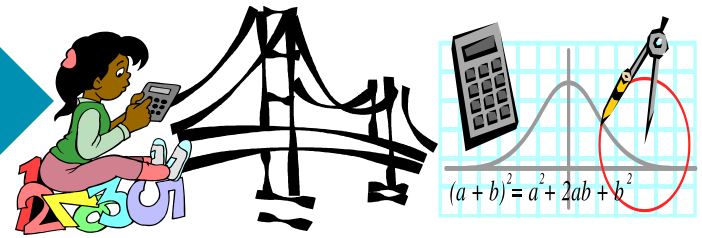


Addition and Subtraction in the CCSS-M for Grades K-4

A quick glance at the progression of expectations for complexity, fluency and representations

	<u>K</u>	<u>1</u>	<u>2</u>	<u>3</u>	
WORD PROBLEMS → See Table 1 for examples of common addition and subtraction situations that students should work with.	<u>K.OA.2:</u> Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	<u>1.OA.1:</u> Add & subtract word problems to 20 with unknowns in all positions. <u>1.OA.2:</u> Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.	<u>2.OA.1:</u> Add & subtract within 100 to solve 1-2 step word problems with unknown in all positions.	<u>3.OA.8:</u> Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	<u>4.OA.3:</u>
<i>Complexity Progression</i>	<i>Simple</i>	<i>Unknowns in all positions</i>	<i>1- and 2-step</i>	<i>2-step</i>	
<i>Concrete-Pictorial-Abstract Progression</i>	<i>Concrete and pictorial representations</i>	<i>Concrete, pictorial and abstract representations</i>	<i>Pictorial and abstract representations</i>	<i>Abstract representations</i>	

Word Problems



COMPLEXITY of word problems

Grade K: simple problems

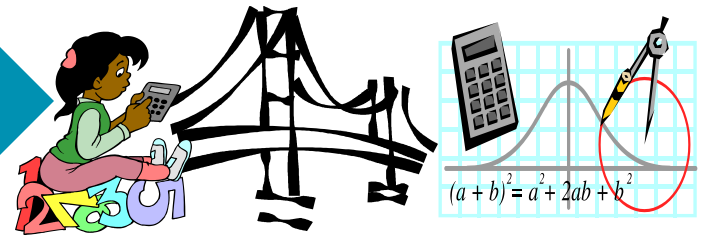
Grade 1: unknowns in all positions

Grade 2: 1 to 2 step

Grade 3: 2-step

Grade 4: multi-step

Word Problems



Numbers to use when working on word problems

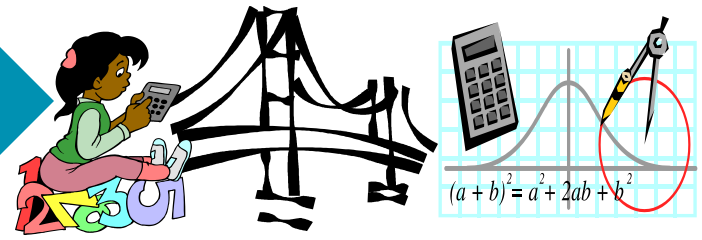
Grade K: up to 10

Grade 1: up to 20

Grade 2: up to 100

**Grade 3 & 4: unspecified but fluency says
(up to 1,000 and 1,000,000 respectively)**

Word Problems



Concrete → Pictorial → Abstract

Grade K: C – P

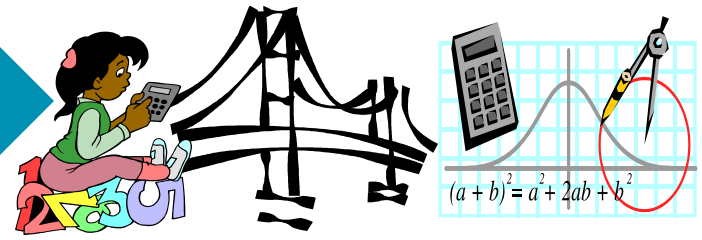
Grade 1: C – P – A

Grade 2: P – A

Grade 3: A

Grade 4: A

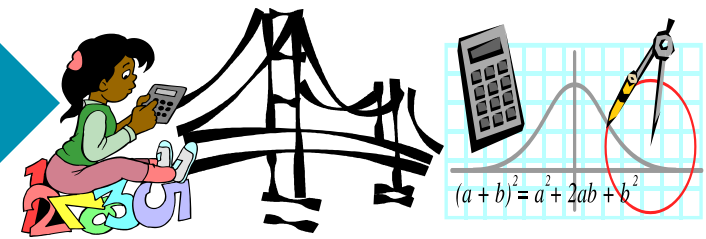
Word Problems



Concrete → Pictorial → Abstract

Please note: Even though in Grade 3 & 4 it says that students should be using abstract ways to solve word problems, this does not mean that you cannot introduce it to the students via concrete or pictorial. We should be working toward helping the students to develop expertise with more efficient strategies.

Equality

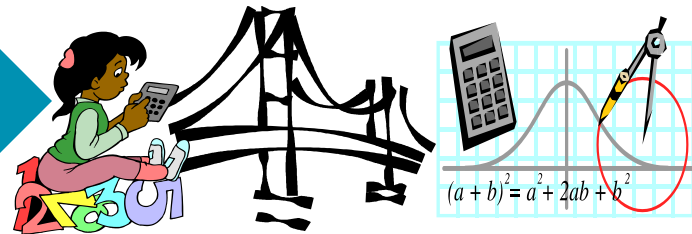


Addition and Subtraction in the CCSS-M for Grades K-4

A quick glance at the progression of expectations for complexity, fluency and representations

	<u>K</u>	<u>1</u>	<u>2</u>
EQUALITY "C/P/A" refers to the types of REPRESENTATIONS students should work with "C" → Concrete Representations "P" → Pictorial Representations "A" → Abstract Representations (i.e., number sentences, expressions, equations)	<p>K.CC.6: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.) (C/P)</p> <p>K.CC.7: Compare two numbers between and 10 presented as written numerals. (A)</p>	<p>1.OA.7: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. (C/P/A)</p> <p>1.NBT.3: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$. (C/P/A)</p>	<p>2.NBT.4: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons. (C/P/A)</p>

Equality



When comparing quantities using $<$, $>$, $=$

Grade K: Numbers to 10. [Orally compares measureable quantities (K.MD.2) & in K.CC.6 & 7]

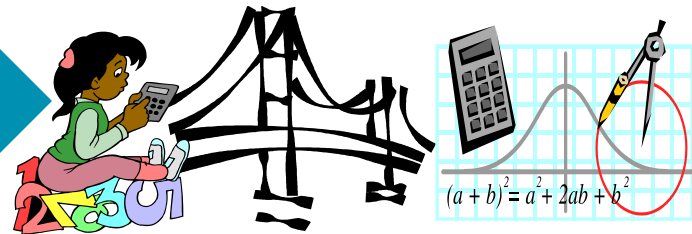
Grade 1: 2-digit numbers

Grade 2: 3-digit numbers

Grade 3: Nothing

Grade 4: up to 1,000,000

Coherence within a grade level



Grade K

Fluency: Add & subtract within 5

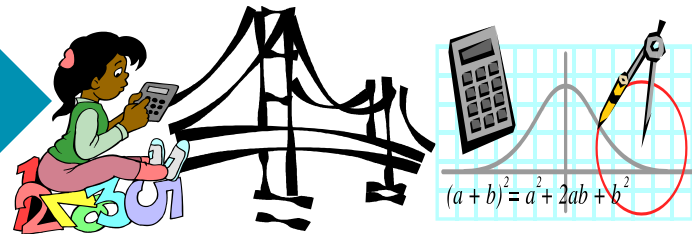
Number Problems: Represent addition & subtraction, develop fluency with finding pairs that make 10 ($C \rightarrow P \rightarrow A$)

Word Problems: Add & subtract within 10 ($C \rightarrow P$)

Comparing Numbers: Identify greater than, less than or equal to ($C \rightarrow P \rightarrow A$)

Base-Ten: Compose & decompose numbers (10 ones and more ones)

Coherence within a grade level



Grade 1

Fluency: Add & subtract within 10

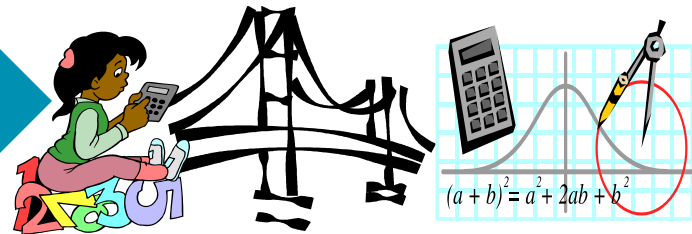
Number Problems: Add & subtract within 20 (A)

Word Problems: Add & subtract within 20 (C → P → A)

Comparing Numbers: 2-digit numbers using <, >, =

Base-Ten : Add (only) within 100 (C → P → A)

Coherence within a grade level



Grade 2

Fluency: Add & subtract within 20

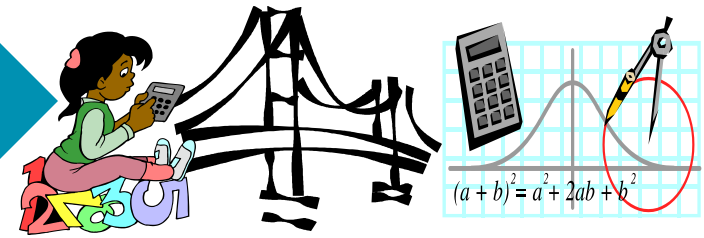
Number Problems: Add & subtract within 100 (A)

Word Problems: Add & subtract within 100 (P → A)

Comparing Numbers: 3-digit numbers using $<$, $>$, $=$

Base-Ten : Add & subtract within 1,000 (C → P / strategies)

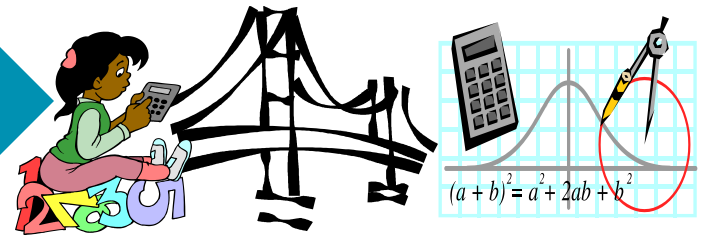
What we hope you'll walk away with



Our Intentions:

- *Increased understanding of the progression of addition and subtraction expectations in grades K-2 (and a little beyond).*
- ***Awareness of available resources for teachers to access on the Standards Toolkit website.***

Phase IV: Incorporation



Some things to look forward to in Phase IV:

- ❖ Website
- ❖ Content webinars
- ❖ K – 2 assessments (tasks and exercises)
- ❖ K – 2 video vignettes, lesson plans & activity sheets
- ❖ Online Professional Collaboration Tool
- ❖ Road Show Symposium: Curriculum Coordinators & Complex Staff (January 2012 – February 2012)



Hawaii Standards Database

PRESENTED BY THE HAWAII STATE DEPARTMENT OF EDUCATION

Home Common Core Standards HCPS III Standards

Welcome

Welcome to the interactive Hawaii Standards Database. This database allows you to search for the information you need to plan for standards-based instruction.

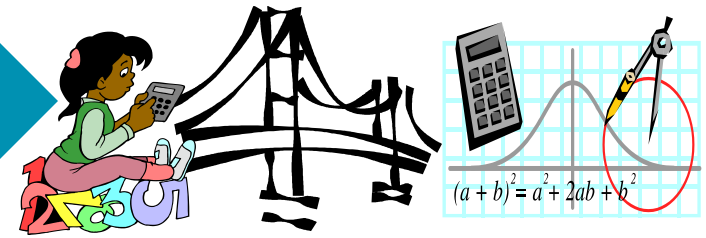
For an overview of the Hawaii Department of Education's Standards-Based Educational resources, please [click here](#).

Governors and state commissioners from 48 states (including Hawaii), the District of Columbia and two territories committed to developing a common core of state standards for proficiency in English language arts and mathematics for grades k-12. The Common Core State Standards (CCSS) for English Language Arts and Literacy in History/Social Studies, Science and Technical Subjects and the CCSS for Mathematics define the knowledge and skills students need to succeed in college and careers when they graduate. In Hawaii, English language arts, teachers in grades k-2, 11-12 and mathematics teachers in grades k-2 and algebra II will begin implementing the CCSS in school year 2011-2012. All teachers will teach the CCSS in school year 2014-2015.



Enter Common Core Standards

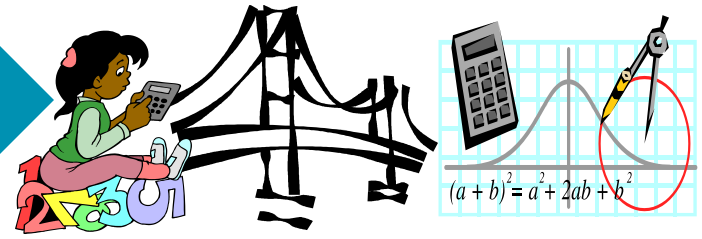
Phase IV: Incorporation



Some things to look forward to in Phase IV:

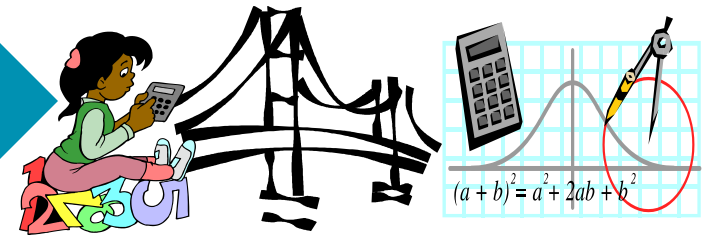
- ❖ Website
- ❖ Content webinars
- ❖ K – 2 assessments (tasks and exercises)
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- ❖ Online Professional Collaboration Tool
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Questions?



**Is there
anything
you are
wondering
about?**

Thank you for joining us!



Aloha!

Sayonara!

Adios!

Auf wiedersehen!

K-den!