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|  | **K** | **1** | **2** | **3** | **4** |
| **STRATEGIES FOR ADDITION AND SUBTRACTION**  “C/P/A” refers to the types of REPRESENTATIONS students should work with  “C” 🡺 Concrete Representations  “P” 🡺 Pictorial Representations  “A” 🡺 Abstract Representations (i.e., number sentences, expressions, equations) | **K.CC.4**: Understand the relationship between numbers and quantities; connect counting to cardinality. (C/P)  **K.OA.1**: Represent addition and subtraction with objects … drawings … expressions and equations (C/P/A)  **K.OA.3**: Decompose numbers less than or equal to 10 into pairs in more than one way… (C/P/A)  **K.OA.4**: For any number from 1 to 9, find the number that makes 10 when added to the given number. (C/P/A)  **K.NBT.1**: Compose & decompose numbers from 11 to 19 into ten ones and some further ones … (C/P/A) | **1.OA.3**: Apply properties of operations as strategies to add and subtract. (A)  **1.OA.4**: Understand subtraction as unknown-addend problem. (C/P/A)  **1.OA.5**: Relate counting to addition & subtraction. (C/P/A)  **1.OA.8**: Determine the unknown number in an addition or subtraction equation relating three whole numbers. (C/P/A)  **1.NBT.4**: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten (C/P/A) | **2.NBT.6**: Add up to four two-digit numbers using strategies based on place value and properties of operations.  **2.NBT.9**: Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) (C/P/A)  **2.NBT.3**: Read & write #s to 1000 by using base 10 numerals, names and expanded form.  **2.NBT.7**: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds . (C/P/A)  **2.OA.4**: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends . (C/P/A) | **3.NBT.2**:  Use place value understanding and properties of operations to perform multi-digit arithmetic. |  |
| *Recommended Strategies* | * Use 5-frames, 10-frames, dot cards and dice to help students quickly recognize pictorial representations of quantities (especially in relation to important benchmark numbers like 5 and 10) | * “Make Ten First” (for addition) or “Get to Ten First” (for subtraction). * Break up a number (according to place value) and add up the parts 🡺 i.e., “partitioning” * Add or subtract 10 (or a multiple of 10) 🡺 begin with “partitioning” then develop expertise with this to be able to do this fluently in one’s mind. | * Break up a number (according to place value) and add up the parts 🡺 i.e., “partitioning” * Subtraction using “compensation” method * Subtraction using regrouping (building toward the use of the traditional “regrouping”/”borrowing” algorithm by breaking up the number according to place value and subtracting the parts (regrouping if necessary) * Associative Property (for sums of up to four 2-digit numbers) | |  |

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| **FLUENCY\* WITH ADDITION AND SUBTRACTION** | **K.OA.5**:  Fluently add and subtract within 5. | **1.OA.6**: Add & subtract within 20, ***demonstrating fluency for addition and subtraction within 10***. Use strategies ...  **1.NBT.5**: Use place value understanding and properties of operations to add and subtract. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.  **1.NBT.6**: Use place value understanding and properties of operations to add and subtract. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. | **2.OA.2**: Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.  **2.NBT.5**: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.  **2.NBT.8**: Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. | **3.NBT.2**:  Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.) | **4.NBT.4**:  Fluently add and subtract multi-digit whole numbers using the standard algorithm. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. A range of algorithms may be used. |
| *Fluency Expectation* | *Up to 5* | *Up to 10* | *Up to 20 and then 100* | *Up to 1,000* | *Up to 1,000,000* |

**\* Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently**.

* Throughout grades K-4, students should have numerous experiences and develop expertise with breaking numbers apart and putting them back together in smart ways.
* Efficient strategies (particularly ones based on place value, making ten or getting to the nearest ten) should be used when “mental math” is indicated.
* At some point during grade 2, students should “know from memory all sums of two one-digit numbers.” This is the only standard (2.OA.2) which expects “memorization and quick recall” regarding addition.
* Students should be supported to develop expertise with more efficient strategies.
* The traditional algorithms for addition and subtraction are expected to be used beginning part of the way through grade 3 and mastered by the end of grade 4.

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| **WORD PROBLEMS**  🡺 See Table 1 for examples of common addition and subtraction situations that students should work with. | **K.OA.2**: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. | **1.OA.1**: Add & subtract word problems to 20 with unknowns in all positions.  **1.OA.2**: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20. | **2.OA.1**: Add & subtract within 100 to solve 1-2 step word problems with unknown in all positions. | **3.OA.8**: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. | **4.OA.3**: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding |
| *Complexity Progression* | *Simple* | *Unknowns in all positions* | *1- and 2-step* | *2-step* | *Multi-step* |
| *Concrete-Pictorial-Abstract Progression* | *Concrete and pictorial representations* | *Concrete, pictorial and abstract representations* | *Pictorial and abstract representations* | *Abstract representations* | *Abstract representations* |

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| **EQUALITY**  “C/P/A” refers to the types of REPRESENTATIONS students should work with  “C” 🡺 Concrete Representations  “P” 🡺 Pictorial Representations  “A” 🡺 Abstract Representations  (i.e., number sentences, expressions, equations) | **K.CC.6**: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.) (C/P)  **K.CC.7**: Compare two numbers between 1 and 10 presented as written numerals. (A) | **1.OA.7**: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. (C/P/A)  **1.NBT.3**: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. (C/P/A) | **2.NBT.4**: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (C/P/A) |  | **4.NBT.2**: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (P/A) |

**Table 1: Common addition and subtraction situations.** Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council, 2009, pp. 32, 33).

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|  | **Result Unknown** | **Change Unknown** | **Start Unknown** |
| **Add to** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?  2 + 3 = ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?  2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?  ? + 3 = 5 |
| **Take from** | Five apples were on the table. I ate two apples. How many apples are on the table now?  5 – 2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?  5 – ? = 3 | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?  ? – 2 = 3 |
|  | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown**  (Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.) |
| **Put Together / Take Apart**  (These “take apart” situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.) | Three red apples and two green apples are on the table. How many apples are on the table?  3 + 2 = ? | Five apples are on the table. Three are red and the rest are green. How many apples are green?  3 + ? = 5, 5 – 3 = ? | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?  5 = 0 + 5, 5 = 5 + 0  5 = 1 + 4, 5 = 4 + 1  5 = 2 + 3, 5 = 3 + 2 |
|  | **Difference Unknown** | **Bigger Unknown** | **Smaller Unknown** |
| **Compare**  (For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.) | (“How many more?” version):  Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?  (“How many fewer?” version):  Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?  2 + ? = 5, 5 – 2 = ? | (Version with “more”):  Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?  2 + 3 = ?, 3 + 2 = ? | (Version with “more”):  Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?  5 – 3 = ?, ? + 3 = 5 |